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Section A

On the optical design of VHE gamma ray imaging Cherenkov telescopes

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Abstract

Optical characteristics of some operational and future VHE atmospheric imaging gamma ray telescopes are discussed. It is shown that due to large longitudinal size of a TeV air shower in the atmosphere, which is comparable to its average distance to a measuring telescope there is a type of optical aberration, that is proportional to diameter of the used reflector (depth of field). The use of imaging telescopes with essentially larger than 10 m diameter reflectors leads to deterioration of image quality which can strongly affect the accuracy of measured image parameters and the gamma–hadron separation power. It is shown that the edges of a reflector of spherical design in the case of large relative aperture A ($A \geq 1$) have to be bent towards the reflectors axis in order to provide the smallest spot size.

1. Introduction

For the past decade the atmospheric Cherenkov imaging technique is used for very high energy (VHE) gamma ray astronomy. During this short time the technique has been significantly improved and has led to the reliable detection of at least four sources of VHE gamma rays. Today some 5 groups and collaborations use imaging telescopes to measure gamma ray sources (Whipple, Crimea, CANGAROO, HEGRA, Durham), more projects are planned or are in the construction phase (see Refs. [1–3]). They use a bundle of photomultiplier tubes (PMT) in the focal plane of a reflector as a coordinate sensitive fast detector and take "snapshots" of extensive air showers in the Cherenkov light. By analysing the shape and orientation of a crude "photo" of a VHE shower it is possible statistically to differentiate between hadron showers and the much rarer showers initiated by gamma rays. Typically the number of PMTs in the focal plane camera of an imaging telescope is between 37 and 300, it covers an angular diameter of 3–5° on the sky with a pixel angular size of 0.1–0.5°. The existing telescopes have energy thresholds in the range 0.2–2.0 TeV. The main difference in the energy threshold comes from the total surface area of a reflector, its reflectivity and the light collection efficiency in the focal plane. Also pixel size and trigger logic affect the energy threshold of a telescope. By a proper choice of a pixel size and a trigger logic one can bias the telescope towards the highest efficiency of gamma ray detection while substantially rejecting hadrons not only at the software but also at the hardware level. Today it is believed that the optimal pixel size α of a TeV imaging telescope, when searching for point sources of gammas, should be in the range of $\alpha \leq (1/4)^\circ$ [4] and $\alpha \geq (1/8)^\circ$ [15] (see also Ref. [16]). If a telescope is also used to observe sources at large zenith angles, where the average distance between the telescope and a shower is larger, the pixel size should be smaller in order to resolve the picture of a distant light distribution. This is also valid when measuring low energy gamma rays in the energy domain of 10–100 GeV, where shower development occurs relatively high in the atmosphere.

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2. Constraints to the largest size of reflector for an imaging Cherenkov telescope

A Cherenkov telescope should allow one to determine accurately the main parameters of showers from the detected light distribution. The optical quality of the reflector and its dimensions are of prime importance. The reflector quality should, in an ideal case, be such as to provide a small (compared to both the pixel and gamma-image size) and constant spot size for a distant point source of light anywhere in the field of view of the camera. In this

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case the "image quality" does not depend on the position of an image in the camera plane. It is known that the spherical design of reflector almost meets these requirements [5] (often it is referred to as the Davies–Cotton design [6]). The main advantages of the spherical design are that one may use mirrors of the same focal length and that it gives a small and nearly constant light spot in the camera field of view of a few degrees. The disadvantage is that due to the shape there is an arrival time spread in the focal plane between the light rays which hit the reflector centre and those that hit the edge.

A parabolic reflector could also be used for the imaging telescopes, although it is inferior in light concentration for off-axis rays compared to the spherical design of the same aperture [5]. Unlike spherical mirror the parabolic one has a tiny spot size at the centre of the field of view for on-axis rays, which rapidly increases for off-axis rays on account of comatic aberration. This makes the images broader, increases the trigger threshold and worsens the "image quality". The parabolic mirror has no time delay due to the shape of the reflector.

It seems that it is also possible to use Schmidt telescopes for the imaging technique. It is well known that a standard or modified Schmidt telescope provides a wide, aberration-free field of view, usually of the order of a few degrees in diameter. The disadvantage of the Schmidt telescope is that it needs a special correction plate of a size comparable to that of the spherical mirror used and it has to be placed at the radius of curvature of the latter. Also, the focal plane detector should be convex. All this makes the construction of large diameter Schmidt telescopes rather complicated and not very realistic. It is worth mentioning that the largest existing Schmidt telescopes are limited to diameters of ~1–1.5 m.

Some articles have recently appeared about the possibility of building large Cherenkov telescopes for the energy range 10–100 GeV. It is believed that this will be a new milestone in gamma ray astrophysics, but here we do not want to dwell upon the details (see for example, Ref. [7]). Among the suggestions to use the existing solar power plants as gamma ray telescopes [8] there are also plans for purpose-built very large imaging gamma ray telescopes. For example in Ref. [7] a system of 9 imaging telescopes each of 100 m diameter reflector is proposed (to limit the spherical aberration, the focal plane detector will "see" only ~0.3 of the diameter of the reflector). Let us have a detailed look at some aspects of the question as to whether there is a physical limitation to the largest size of reflector for an imaging telescope.

To parameterise the measured Cherenkov light distribution of an extensive air shower in the focal plane of an imaging telescope the formalism of second moments proposed by Hillas [9] is used. The main parameters of an elliptical image in the camera plane are its width and length, characterising the shape of a shower and the azimuthal angle alpha, showing orientation of the main

Table 1
The theoretical and experimental average values of the shape parameters for the images of proton showers are shown, together with the theoretical value for gammas.

Authors and references	Particle type	Length [deg]	Width [deg]
A.M. Hillas, [9] Monte Carlo	gamma	0.3	0.1
	proton	0.5	0.3
T.C. Weekes, [10] experiment	proton	0.44–0.48	0.24–0.28
V.M. Vladimirsky, [11], experiment	proton	0.42	0.31

axis of the ellipse with respect to the line which connects its centroid to the telescope target position. In Table 1 the theoretical and experimental average values of the shape parameters for the images of proton showers are shown, together with the theoretical value for gammas. From Table 1, the characteristic image size of a gamma ray is ~0.2° and for a proton it is ~0.4°. It should be mentioned that the experimental values quoted above are rather sensitive to the "tail cut" applied to the images (the minimum signal after subtraction of different kinds of noise, which is still taken into account in the calculation of the second moments), therefore they should be considered as guideline values. One may conclude that the difference in size between an image of a gamma and that of a proton is on the scale of 0.1–0.2° and it is obvious that the sum of all optical aberrations and distortions of a reflector must be less than this value. Only then can one reconstruct reliably the shape and orientation of an air shower image. Any effect which will smear out an image by the above amount or even more, will make difficult or practically impossible further discrimination of hadrons.

Unlike telescopes of optical observatories, which measure light from objects "at infinity", a Cherenkov telescope takes "photos" of showers which develop in the atmosphere mainly between 4 and 10 km above sea level for the TeV energy range. The light from different heights (i.e. different distances from the telescope) is brought to the plane of the camera and since this does not correspond to the "best" focal plane, the images are smeared out. Let us estimate the scale of this effect.

Let us assume for simplicity that the optical system of a telescope is aberration free. If the camera plane coincides with the focal plane of the used reflector, then the "best" image plane of a part of a shower at a distance L from the telescope will be shifted from the focal plane by a distance of

$$\delta = f^2/(L+f),$$

where f is the focal length of the reflector (see Fig. 1). The linear extent of the light spot will be equal to

$$\delta \cdot D/f = D \cdot f/(L+f),$$

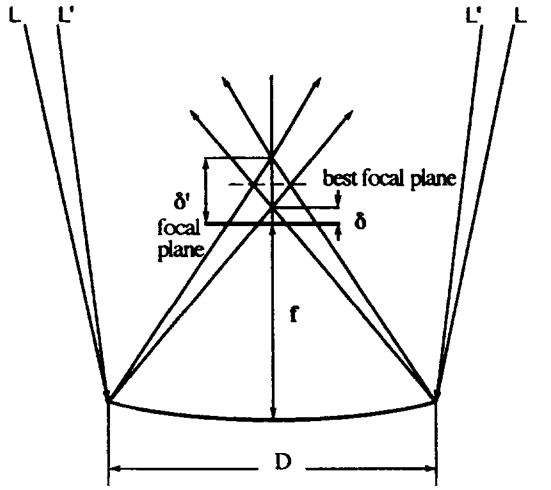

Fig. 1. The "best" image planes for sections of a shower at distances L and $L'(L > L')$ from the telescope will be shifted from the focal plane by distances of δ and δ'.

where D is the diameter of the reflector. Correspondingly the angular diameter θ will be equal to

$$\theta = D/L ,$$

because $f \ll L$. One can see that the angular size of the light spot does not depend on the focal length f, but only on the diameter D of the reflector.

One may shift the camera to some chosen, optimal position, away from the focal plane, but still only for a restricted region of shower height will the image be sharp. Due to large size of longitudinal development of a TeV shower (see, for example, Ref. [12]), the measured light distribution will be smeared out by the characteristic scale factor of D/L. The image of a shower in the camera plane is nearly elliptical and its major axis displays the longitudinal development of the shower. If the camera plane is shifted so as to provide a sharp image from the region of shower maximum (this is a usual procedure in the optical adjustment of the Crimean and the HEGRA telescopes), the edges of the ellipse aligned with the major axis, depending on the diameter of a telescope will be more or less smeared out. If a telescope performs observations at small zenith angles, at which main bulk of the data is usually recorded (this is true for gamma ray sources which culminate not very far away from the local zenith), with the increase of zenith angle the shower maximum moves away from the telescope thus shifting again the best image plane. Also, when measuring gammas in the energy range of 10–100 GeV, the shower maximum is reached higher in the atmosphere and the longitudinal development of a shower is shorter compared to a TeV gamma one. Therefore an optimal solution must be found in each case.

To illustrate how large a light spot can be due to this type of aberration (assuming ideal, aberration free-optics) we have made estimates for $D = 30$ m (an imaginary telescope), $D = 10$ m (a Whipple-type telescope) and $D = 3$ m (as for the first telescope of the HEGRA collaboration [13]). Estimates show that the angular size of a light spot could be as large as ~15′ for the 30 m diameter one, ~5′ for the 10 m telescope and ~1.5′ for the 3 m telescope.

So the main conclusion is that the use of imaging telescopes with essentially larger, more than 10 m diameter reflectors leads to a deterioration of image quality which can strongly affect the accuracy of measured image parameters.

3. On methods of improving the image quality of VHE gamma ray telescopes and the optimal shape of a reflector

Aberrations of a concave mirror are proportional to relative aperture of it, which is defined as the ratio of diameter D to focal length f. Therefore to construct a telescope which provides a high quality image in the focal plane it is necessary to use a reflector with small relative aperture. The light collection ability of a telescope is determined by the reflector surface area and reflectivity and therefore the diameter D is usually determined from the physical task. If the diameter D is given, the use of a very long focal length is in practice very difficult, because the diameter of a focal plane detector scales up with the focal length and the mechanical construction should be very strong in order to hold the camera and the masts without substantial bending in the focal plane. In this case problems may arise also due to the necessity to provide a heavy counterweight on the side of mirrors for balancing the camera and the masts. Fortunately there is another solution of this problem, which is used in the construction of telescopes. Nearly all Cherenkov telescopes are constructed from a large number of small diameter mirrors. Each mirror has a small value of relative aperture which provides a high quality image. Also, the total price of mirrors of such a tessellated reflector is much lower compared to that of a single mirror of the same reflecting surface.

Aberrations of a mirror also are proportional to incident angle of light, which is defined as the angle between the optical axis of the former and the direction of incident beam. Mirrors of a tessellated reflector are placed on a concave surface and they are all adjusted so as to collect parallel to the optical axis of the reflector beam of light into a single point in the focal plane. Therefore the mirrors at the edge of a concave surface are set inclined towards the axis of the reflector and they work under large incident angles even in the case of parallel to the optical axis of reflector beam of light.

Let us try to estimate aberration of a single spherical mirror component of a tessellated reflector of focal length f. For a quick estimate of the angular size of a blur ϕ of

such a mirror, inclined to the optical axis of the whole optical system at an angle ω one may use the following expression (see, for example, Ref. [17]):

$$\phi = A_0^3/64 + 3 \cdot A_0^2 \cdot \omega/16 + A_0 \cdot \omega^2/2 \qquad (1)$$

where $A_0 = d/f$ is the relative aperture of a single element of the optical system. The first, second, and the third terms in Eq. (1) give correspondingly the spherical, comatic and astigmatic aberrations. It should be mentioned that this formula gives the blur size for the image plane at a distance f from the mirror centre. When moving this plane along the optical axis one may find another position where the blur size is less.

Using Eq. (1) one may calculate that the angular diameter of a blur spot of a single mirror at the edge of the 10 m diameter reflector of Whipple telescope is not less than 0.3° for a light source at infinity. The blur-spot size of a single mirror of the Crimean Astrophysical Observatory telescope is ~0.19°. A single mirror at the edge of the 3 m diameter reflector of the first HEGRA telescope (the reflector includes 18 mirrors each with a diameter of 0.6 m) gives a blur-spot size of 0.10° and for the 4.3 m diameter reflector of the 2nd HEGRA telescope (includes 30 mirrors of the same type) it gives a blur-spot size of 0.18°. These correspond to the widest individual blur sizes as the mirrors from the outer ring of reflectors have the largest aberration. The mirrors which are closer to the optical axis have smaller aberrations and the blur-spot size of a reflector is a result of superposition of focused light from all mirrors on it. As the blur size of an optical system can be calculated only in simple, limited cases including mainly a single optical element, it is difficult to obtain in this way the resulting spot size for a tessellated reflector and one may use Eq. (1) as a conservative, i.e., large, estimate of the image blur of a tessellated reflector.

From Eq. (1) and above mentioned simple considerations one may draw useful conclusions about the ways which may help to obtain a small spot size for a telescope:

1) For a given f one may use a reflector with small diameter. This leads to a high energy threshold, but one may use several telescopes in parallel, summing up the outputs. In this case one needs multiple focal plane detectors and electronics for the data acquisition.

2) For a given f and a diameter D of a tessellated reflector one may use individual mirrors of small diameters (this helps to obtain a small spot size for on-axis rays; for off-axis rays this helps up to a small incident angle θ after which the global aberration, which is due to the overall shape of the reflector, dominates the image shape [5]). In this case one needs large number of mirrors and mechanical adjustment units.

3) For a given diameter D one may use individual mirrors of long focal lengths. In this case one needs a strong mechanical construction of the reflector and of the masts in order to hold the camera in the focal plane. Also, both the camera and the light sensor size should be scaled up.

To obtain more detailed and precise information about the point spread function of above mentioned telescopes we have developed a ray tracing program. For a given telescope using this program we found the minimum spot size and the position of each mirror on its reflector.

We have applied this program to the reflectors of the Whipple, the Crimean and the HEGRA telescopes. In Fig. 2 are presented the results of calculations of the proportion of focused light which falls inside a circle of a given angular diameter for the Whipple telescope. The calculations have been performed for 4 values of incident angle of light between 0° and 1.5°. It should be mentioned, that here we have used a reflector shape somewhat different from that of Ref. [5]. For a source at infinity we calculated the rms deviation of light distribution in the focal plane and, while varying the position and distance of each mirror, found the minimum of it. In this way we found the optimal shape of the reflector and the position of each mirror on it.

In Ref. [5] calculations were performed for the Whipple telescope under the simple assumption of the third-order aberration theory that the disk of least confusion due to astigmatic aberration takes place about midway between the sagittal and tangential line foci and therefore all the mirrors are set out on a spherical frame with a radius equal to the focal length of mirrors.

Comparison of our result with the spherical design of Ref. [5] (see Fig. 3) shows that some difference arises only for the mirrors which are close to the edge of the reflector and work under large incident angles. One may assume that the "focal length" (the distance from the mirror at which the smallest spot lies) of a mirror is effectively reduced when working under large incident angles of light (this effect is briefly mentioned in the article of Davies and Cotton). In Table 2 is shown the shortening Δ of the "focal length" of a mirror with respect to the focal length f of a spherical frame depending on the distance ρ of the

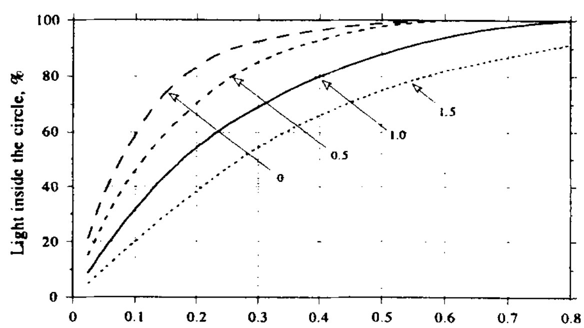

Fig. 2. The results of calculations of percentage of total light which is inside a circle of a certain angular diameter for the 10 m Whipple telescope.

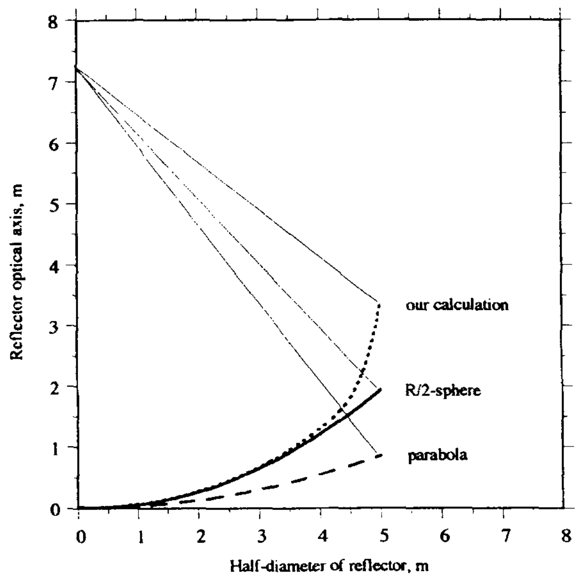

Fig. 3. Comparison between the best reflector shape according to our result and the spherical design.

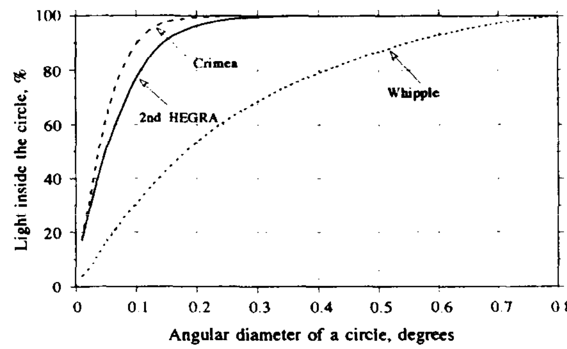

Fig. 4. The integral light collection characteristics of the Whipple, the Crimean and the 2nd HEGRA telescopes for the incident angle of 1°.

mirror from the axis of the reflector (or its working angle θ) for the parameters of reflector of the Whipple telescope.

The results of our simulation are in good agreement with the measured parameters of the Whipple telescope. From Fig. 2 one may see that in the circle of 0.25° diameter 90% of the incident light is collected only in the case of parallel to the telescope axis beam of light. With increase of angle of incidence the fraction of light in the circle decreases and for the incident angle of 1.5° only 50% of light is collected there. One may conclude that the optical design of this telescope is satisfactory for the field of view of radius ~1.5° and for the pixel size of 0.25°.

In Fig. 4 one may see the simulated integral light collection characteristics of the Whipple, the Crimean and the 2nd HEGRA telescopes for the incident angle of 1°. The choice of this angle is based on the fact that the greatest proportion of showers detected by ~3° acceptance telescopes have light spot centroids at ~1° distance from the camera centre. One can see that the Crimean and HEGRA telescopes have superior optical characteristics, but their pixel size of ~0.4° does not permit exploitation of all of the advantages of these reflectors. The next telescopes of the HEGRA system will have a pixel size of 0.24° (since the beginning of 1995 the first telescope has been running with a 127-pixel camera of pixel size 0.24°).

In Table 3 the main optical parameters of some of the above mentioned telescopes are presented. Here A_{mirror} is the mirror surface area of a telescope, (2 × rms) deviation is the calculated characteristic diameter of the light spot and the "90%" is the angular diameter of the light spot, where 90% of the incident light is collected.

It is interesting to note, that the experimentally measured and simulated by ray tracing method point spread functions for the above mentioned telescopes give values ~2 times less than the blur size estimated from the simple formula (1) for a single mirror at the edge of a reflector. For example, the measured point spread function of the

Table 3
The main optical parameters calculated by ray tracing method for the Crimean, the 2nd HEGRA and the Whipple telescopes are presented. Here A_{mirror} is the mirror surface area of a telescope, (2 × rms) deviation is the calculated characteristic diameter of the light spot and the "90%" is the angular diameter of the light spot, where 90% of the incident light is collected.

Telescope	A_{mirror} [m^2]	(2 × rms) [deg]	90% of light [deg]
Crimea, 1 module	4.5	0.045	0.08
Total on 1 mount	27	0.045	0.08
2nd HEGRA	10 (8.5)	0.058	0.13
Whipple	75	0.203	0.55

Table 2
Here is shown the shortening Δ of the "focal length" of a mirror with respect to the focal length f of a spherical frame depending on the distance ρ of the mirror from the axis of the reflector (or its working angle θ) for the parameters of reflector of the Whipple telescope.

ρ [cm]	0	50	100	150	200	250	300	350	400	450	500
θ [deg]	0	2	4	6	8	10	12	14	16	18	20
Δ [cm]	0	0	0	<1	<1	1.5	3	5	9	20	95

first HEGRA telescope is ~3.6′ [13], while the formula (1) gave 6′ (=0.1°). Taking into account the type of aberration due to the diameter of the telescope, one finds that the total aberration of this telescope is ~4′.

As far as we know, neither in operation nor in project is there a second generation telescope allowing such high quality imaging.

It is evident that small diameter telescopes are superior in their optical resolution and they can be used to realise the idea of Hillas to get a resolution of a few arc minutes [14].

4. Conclusions

It is shown that due to the large size of longitudinal development of a TeV air shower in the atmosphere and its finite distance to a measuring telescope there is a "depth of field" type aberration, which is proportional to the diameter of the used reflector. Therefore the large diameter reflectors have low angular resolution and conversely, the small diameter ones have a high resolution. The use of imaging telescopes with essentially larger than 10 m diameter reflectors leads to deterioration of image quality which can strongly affect the accuracy of measured image parameters and make the gamma–hadron separation problematic.

In the design of a reflector of spherical shape ($R/2$-sphere, where R is the radius of curvature) of large relative aperture A ($A \geq 1$), to achieve the best point spread function one has to take into account the distinction of the reflector shape from the spherical one at the edges, where it should be more bent towards the axis of the reflector. Because this will increase the arrival time spread in the focal plane, the alternative solution will be, for a given diameter D of a reflector to choose a focal length F so as to fulfil the condition $D/F = A \leq 1$, when the shortening of the "focal length" for an edge mirror is not yet substantial.

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